J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

MATRICES + SYJC TEST – 01 – SET 1

TOPIC : CORRELATION DURATION – 1 $^1/_2$ HR MARKS – 40

SOLUTION SET

SECTION - I

Q-1

01. $X + Y = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$

Find the matrix X

SOLUTION

$X + Y = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$

$X - Y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$

$2X = \begin{pmatrix} 8 & 8 \\ 2 & 2 \end{pmatrix}$

$X = \frac{1}{2}\begin{pmatrix} 8 & 8 \\ 2 & 2 \end{pmatrix}$

$X = \begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix}$

02. $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & a & 2 \\ 5 & 7 & 3 \end{pmatrix}$ is a singular matrix
Find a

SOLUTION

Since A is singular matrix

$|A| = 0$

$1(3a - 14) - 2(6 - 10) + 3(14 - 5a) = 0$

$3a - 14 - 2(-4) + 42 - 15a = 0$

$3a - 14 + 8 + 42 - 15a = 0$

$36 - 12a = 0$

$\therefore\ a = 3$

03. if $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ Find $|AB|$

SOLUTION

AB

$= \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 + 3 & 2 + 4 \\ 2 + 6 & 4 + 8 \end{pmatrix}$

$= \begin{pmatrix} 4 & 6 \\ 8 & 12 \end{pmatrix}$

$|AB| = 48 - 48 = 0$

04. find the ADJOINT of the matrix

$A = \begin{pmatrix} 2 & -3 \\ 3 & 5 \end{pmatrix}$

COFACTOR'S

$A_{11} = (-1)^{1+1}\, 5 = 5$

$A_{12} = (-1)^{1+2}\, 3 = -3$

$A_{21} = (-1)^{2+1}\, (-3) = 3$

$A_{22} = (-1)^{2+2}\, 2 = 2$

COFACTOR MATRIX OF A

$= \begin{pmatrix} 5 & -3 \\ 3 & 2 \end{pmatrix}$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$= \begin{pmatrix} 5 & 3 \\ -3 & 2 \end{pmatrix}$

Q-2

Q2. Attempt any TWO of the following (3 marks each)

01.

$$\left\{2\begin{pmatrix}5 & 0 & -1\\ 1 & 2 & -3\end{pmatrix} - 3\begin{pmatrix}2 & -1 & 1\\ -4 & 2 & 3\end{pmatrix}\right\}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$$\left\{\begin{pmatrix}10 & 0 & -2\\ 2 & 4 & -6\end{pmatrix} - \begin{pmatrix}6 & -3 & 3\\ -12 & 6 & 9\end{pmatrix}\right\}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}10-6 & 0+3 & -2-3\\ 2+12 & 4-6 & -6-9\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}4 & 3 & -5\\ 14 & -2 & -15\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}4+3-0\\ 14-2-0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}7\\ 12\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

By equality of two matrices

$x = 7$ & $y = 12$

02. $x + y + z = 6$

$3x - y + 3z = 10$

$5x + 5y - 4z = 3$

$AX = B$

$$\begin{pmatrix}1 & 1 & 1\\ 3 & -1 & 3\\ 5 & 5 & -4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\10\\3\end{pmatrix}$$

$R_2 - 3R_1$

$$\begin{pmatrix}1 & 1 & 1\\ 0 & -4 & 0\\ 5 & 5 & -4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-8\\3\end{pmatrix}$$

$R_3 - 5R_1$

$$\begin{pmatrix}1 & 1 & 1\\ 0 & -4 & 0\\ 0 & 0 & -9\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-8\\-27\end{pmatrix}$$

$$\begin{pmatrix}x + y + z\\ -4y\\ -9z\end{pmatrix} = \begin{pmatrix}6\\-8\\-27\end{pmatrix}$$

BY EQUALITY OF TWO MATRICES

$-9z = -27$

$z = 3$ …….. (1)

$-4y = -8$

$y = 2$ …….. (2)

$x + y + z = 6$

$x + 2 + 3 = 6$

$x = 1$ …….. (3)

SS : { 1 , 2 , 3 }

03. $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ Show that : $A^2 - 5A + 7I = 0$

Hence find : A^{-1}

Q-2

STEP 1

$$LHS = A^2 - 5A + 7I$$

$$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} - 5 \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 7 & 3 - 10 + 7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = RHS$$

STEP 2 SOLVING FOR A^{-1}

$|A| = 6 + 1 = 7 \neq 0$ Hence A^{-1} exists

$A^2 - 5A + 7I = 0$

$A^2 A^{-1} - 5A A^{-1} + 7 IA^{-1} = 0$

$AA A^{-1} - 5A A^{-1} + 7 IA^{-1} = 0$

$AI - 5I + 7A^{-1} = 0$

$A - 5I + 7A^{-1} = 0$

$7A^{-1} = 5I - A$

$$7A^{-1} = 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$7A^{-1} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$7A^{-1} = \begin{pmatrix} 5 - 3 & 0 - 1 \\ 0 + 1 & 5 - 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{7} \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

Q3. Attempt any TWO of the following (4 marks each)

01. $A = \begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{pmatrix}$

$$\begin{aligned} |A| &= 1(-0-3) - 2(0+1) - 2(0-2) \\ &= 1(-3) - 2(1) - 2(-2) \\ &= -3 - 2 + 4 \\ &= -1 \\ &\neq 0 \quad \text{Hence } A^{-1} \text{ exists} \end{aligned}$$

$AA^{-1} = I$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_3 + R_1$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

R_{23}

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_2 + 2R_3$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_1 - 2R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_3 + 2R_2$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$R_1 + 2R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$I.\,A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

Q - 3

02. $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$

COFACTOR'S

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 1(4 - 4) = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -1(-4 - 2) = 6$

$A_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = 1(-2 - 1) = -3$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = -1(8 - 6) = -2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1(4 - 3) = 1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = -1(2 - 2) = 0$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1(4 - 3) = 1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -1(2 + 3) = -5$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1(1 + 2) = 3$

COFACTOR MATRIX OF A

$$= \begin{pmatrix} 0 & 6 & -3 \\ -2 & 1 & 0 \\ 1 & -5 & 3 \end{pmatrix}$$

ADJ A

= TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{pmatrix}$$

|A|

$= 1(4 - 4) - 2(-4 - 2) + 3(-2 - 1)$

$= 1(0) - 2(-6) + 3(-3)$

$= 0 + 12 - 9$

$= 3$

$\mathbf{A^{-1}} = \dfrac{1}{|A|} \cdot \text{adj } A$

$$= \frac{1}{3} \begin{pmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{pmatrix}$$

03. $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & a \\ 4 & b \end{pmatrix}$ **Q-3**

such that :$(A + B)^2 = A^2 + B^2$,find a & b

SOLUTION

GIVEN

$(A + B)^2 = A^2 + B^2$

$(A + B)(A + B) = A^2 + B^2$

$A^2 + AB + BA + B^2 = A^2 + B^2$

$AB + BA = 0$

$$\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 4 & b \end{pmatrix} + \begin{pmatrix} 1 & a \\ 4 & b \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 - 4 & a - b \\ 2 - 4 & 2a - b \end{pmatrix} + \begin{pmatrix} 1 + 2a & -1 - a \\ 4 + 2b & -4 - b \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & a - b \\ -2 & 2a - b \end{pmatrix} + \begin{pmatrix} 1 + 2a & -1 - a \\ 4 + 2b & -4 - b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 + 1 + 2a & a - b - 1 - a \\ -2 + 4 + 2b & 2a - b - 4 - b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 + 2a & -b - 1 \\ 2 + 2b & 2a - 2b - 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

By equality of matrices

$-2 + 2a = 0$ $\therefore$ $a = 1$
$-b - 1 = 0$ $\therefore$ $b = -1$

END OF SECTION - I

SECTION - II

Q-4

Q4.1. $n = 100$; $\overline{x} = 62$; $\overline{y} = 53$;

$\sigma x = 10$; $\sigma y = 12$

$\Sigma(x - \overline{x})(y - \overline{y}) = 8000$

SOLUTION

$$r = \frac{cov(x,y)}{\sigma x \, . \, \sigma y}$$

$$= \frac{\frac{\Sigma(x - \overline{x})(y - \overline{y})}{n}}{\sigma x \, . \, \sigma y}$$

$$= \frac{\frac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

02.

the ranks of same 10 students in Mathematics and Physics are as follows . Two numbers within brackets denote the ranks of students in Mathematics an Physics

(1,1) ; (2,10) ; (3,3) ; (4,4) ; (5,5) ; (6,7) ; (7,2) ; (8,6) ; (9,8) ; (10,9) .

Calculate rank correlation

SOLUTION

x	y	d = |x − y|	d^2
1	1	0	0
2	10	8	64
3	3	0	0
4	4	0	0
5	5	0	0
6	7	1	1
7	2	5	25
8	6	2	4
9	8	1	1
10	9	1	1
			$\Sigma d^2 = 96$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(96)}{10(100 - 1)}$$

$$= 1 - \frac{6(96)}{10(99)}$$

$$= 1 - \frac{32}{55}$$

$$= \frac{23}{55}$$

$$= 0.42$$

03. coefficient of correlation between variables X and Y is 0.3 and their covariance is 12 . The variance of X is 9 . Find standard deviation of Y

SOLUTION

$r = 0.3$, $cov(x,y) = 12$, $\sigma x^2 = 9$,

$$r = \frac{cov(x,y)}{\sigma x \, . \, \sigma y}$$

$$0.3 = \frac{12}{3 \times \sigma y}$$

$$\sigma y = \frac{12}{3 \times 0.3}$$

$$= \frac{120}{3 \times 3}$$

$$= \frac{40}{3} = 13.33$$

04. The coefficient of rank correlation is 0.6 . If the sum of the squares of the difference in ranks is 66 , find the number of students in the group

SOLUTION

$R = 0.6 \quad ; \quad \Sigma d^2 = 66$

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.6 = 1 - \frac{6(66)}{n(n^2 - 1)}$$

$$\frac{6(66)}{n(n^2 - 1)} = 1 - 0.6$$

$$\frac{6(66)}{n(n^2 - 1)} = 0.4$$

$$\frac{6(66)}{n(n^2 - 1)} = \frac{4}{10}$$

$$n(n^2 - 1) = \frac{6 \times 66 \times 10}{4}$$

$$n(n^2 - 1) = 990$$

$$(n - 1).n.(n + 1) = 9.10.11$$

On comparing

$n = 10$

Q-5

Q5.1. find number of pair of observations from the following data

$r = 0.4 \quad ; \quad \Sigma xy = 108 \quad ; \quad SDy = 3 \quad ; \quad \Sigma x^2 = 900$; where x and y are deviations from their respective means

SOLUTION

$r = 0.4 \quad ; \quad \Sigma(x - \overline{x})(y - \overline{y}) = 108 \quad ;$

$\sigma y = 3 \quad ; \quad \Sigma(x - \overline{x})^2 = 900$

$$\sigma y = 3$$

$$\sqrt{\frac{\Sigma(y - \overline{y})^2}{n}} = 3$$

$$\sqrt{\Sigma(y - \overline{y})^2} = 3\sqrt{n} \quad ..... \quad (1)$$

$$r = \frac{\Sigma(x - \overline{x}).(y - \overline{y})}{\sqrt{\Sigma(x - \overline{x})^2}\sqrt{\Sigma(y - \overline{y})^2}}.$$

$$0.4 = \frac{108}{\sqrt{900} \,.\, 3\sqrt{n}} \quad \text{....... From (1)}$$

$$\frac{4}{10} = \frac{108}{30 \,.\, 3\sqrt{n}}$$

$$\sqrt{n} = \frac{108 \times 10}{30 \times 3 \times 4}$$

$$\sqrt{n} = 3$$

Squaring ;

$$n = 9$$

02

$N = 8 \; ; \quad \Sigma x = 56 \; ; \quad \Sigma y = 40 \quad ; \quad \Sigma x^2 = 524$

$\Sigma y^2 = 256 \; ; \quad \Sigma xy = 364$

SOLUTION

$$r = \frac{n\Sigma xy - \Sigma x.\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$r = \frac{8(364) - 56(40)}{\sqrt{8(524) - 56^2}\sqrt{8(256) - 40^2}}$$

$$r = \frac{2912 - 2240}{\sqrt{4192 - 3136}\sqrt{2048 - 1600}}$$

$$r = \frac{672}{\sqrt{1056}\sqrt{448}}$$

$$r = \frac{672}{\sqrt{1056 \times 448}}$$

$$\log r = \log 672 - \frac{1}{2}\left(\log 1056 + \log 448\right)$$

$$\log r = 2.8274 - \frac{1}{2}\left(3.0237 + 2.6513\right)$$

$$\log r = 2.8274 - \frac{1}{2}\, 5.6750$$

$$\log r = 2.8274 - 2.8375$$

$$\log r = \overline{1}.9899$$

$$r = AL\,(\overline{1}.9899)$$

$$r = 0.9770$$

03. the coefficient of rank correlation of marks obtained by 10 students in English and Economics was found to be 0.5. It was later found that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 3 instead of 7 . Find the correct coefficient of rank correlation

SOLUTION

N = 10 , R = 0.5

Incorrect d = 3 while correct d = 7

STEP – 1

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(100 - 1)}$$

$$0.5 = 1 - \frac{6\Sigma d^2}{10(99)}$$

$$0.5 = 1 - \frac{\Sigma d^2}{165}$$

$$\frac{\Sigma d^2}{165} = 1 - 0.5$$

$$\frac{\Sigma d^2}{165} = 0.5$$

$$\Sigma d^2 = 82.5$$

STEP 2

$$\Sigma d^2 = 82.5$$

-3^2 -9

         $+40$

$+7^2$ $+49$

$$\Sigma d^2_{correct} = 122.5$$

STEP 3

$$R_{correct} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(100 - 1)}$$

$$= 1 - \frac{6(122.5)}{10(99)}$$

$$= 1 - \frac{245}{330}$$

$$= 1 - \frac{49}{66}$$

$$= \frac{17}{66}$$

$$= 0.2575$$

Q-5

Q-6

Q6 .1

x : 3 4 6 7 10

y : 9 11 14 15 16 . Find Karl Pearson's Correlation coeff.

SOLUTION

x	y	$x-\overline{x}$	$y-\overline{y}$	$(x-\overline{x})^2$	$(y-\overline{y})^2$	$(x-\overline{x})(y-\overline{y})$
3	9	−3	−4	9	16	12
4	11	−2	−2	4	4	4
6	14	0	1	0	1	0
7	15	1	2	1	4	2
10	16	4	3	16	9	12
30 Σx	65 Σy	0 $\Sigma(x-\overline{x})$	0 $\Sigma(y-\overline{y})$	30 $\Sigma(x-\overline{x})^2$	34 $\Sigma(y-\overline{y})^2$	30 $\Sigma(x-\overline{x})(y-\overline{y})$
$\overline{x} = 6$ $\overline{y} = 13$						

$$r = \frac{\Sigma(x-\overline{x}).(y-\overline{y})}{\sqrt{\Sigma(x-\overline{x})^2}\sqrt{\Sigma(y-\overline{y})^2}}.$$

$$r = \frac{30}{\sqrt{30} \times \sqrt{34}}$$

$$r = \frac{30}{\sqrt{30 \times 34}}$$

taking log on both sides

$$\log r = \log 30 - \frac{1}{2}\left(\log 30 + \log 34\right)$$

$$\log r = 1.4771 - \frac{1}{2}[1.4771 + 1.5315]$$

$$\log r = 1.4771 - \frac{1}{2}(3.0086)$$

$$\log r = 1.4771 - 1.5043$$

$$\log r = \overline{1}.9728$$

$$r = AL(\overline{1}.9728)$$

$$r = 0.9393$$

02. Given X : 1 2 3 …………. n Show that : $Cov(x,y) = \frac{n^2 - 1}{12}$

Y : 1 2 3 ………… n

SOLUTION

$$Cov(x,y) = \frac{\Sigma xy}{n} - \overline{x}.\overline{y}$$

STEP 1 : Find means $\overline{x}$ & $\overline{y}$

$$\Sigma x = 1 + 2 + 3 + \ldots\ldots n = \frac{n(n+1)}{2}$$

$$\overline{x} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\Sigma y = 1 + 2 + 3 + \ldots\ldots n = \frac{n(n+1)}{2}$$

$$\overline{y} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

STEP 2 : $\frac{\Sigma xy}{n}$

$$= \frac{1.1 + 2.2 + 3.3 + \ldots\ldots + n.n}{n}$$

$$= \frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n}$$

$$= \frac{n(n+1)(2n+1)}{6n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

STEP 3 : $cov(x,y) = \frac{\Sigma xy}{n} - \overline{x}.\overline{y}$

$$= \frac{(n+1)(2n+1)}{6} - \frac{n+1}{2}.\frac{n+1}{2}$$

$$= \frac{n+1}{2}\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)$$

$$= \frac{n+1}{2}\left(\frac{4n+2-3n-3}{6}\right)$$

$$= \frac{n+1}{2}\,\frac{n-1}{6}$$

$$= \frac{n^2-1}{12}$$

03. **Q-6**

SOLUTION

X : Marks in History

Y : Marks in Geography

X History	Y Geog.	x	y	d $=\|x-y\|$	d^2
*70	# 80	1.5	1.5	0	0
*70	60	1.5	5	3.5	12.25
65	# 80	3	1.5	1.5	2.25
60	70	4	3	1	1
55	65	5	4	1	1
50	50	6	6	0	0
40	42	7	7	0	0
30	28	8	8	0	0
					$\Sigma d^2 = 16.50$

CORRECTION FACTORS

Using $\frac{m(m^2-1)}{12}$

$$T_x = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = 0.5$$

$$T_y = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = 0.5$$

$\mathbf{\Sigma d^2}$ **corrected** $= \Sigma d^2 + Tx + Ty$

$= 16.50 + 0.5 + 0.5$

$= 17.50$

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(17.50)}{8(64-1)}$$

$$= 1 - \frac{6(17.50)}{8(63)}$$

$$= 1 - \frac{35}{168}$$

$$= \frac{133}{168}$$

$$= 0.7918$$

log 133 − log 168

  2.1239
− 2.2253

AL $\bar{1}.8986$